

## Arithmetic

### Proportional Parts:

This is dividing quantities in a given ratio.

Rules:

1. Add the ratio's to get the total of parts.
2. Divide the quantity by the total in part 1.
3. Multiply each ratio by answer in part 2.

#### Example 1:

- Divide
- (i) €32 in the ratio of 5 : 3
  - (ii) 63 cm in the ratio  $\frac{1}{2} : 1 : 2$

**Answer:**

- |   |   |
|---|---|
| <p>(i)</p> <ol style="list-style-type: none"> <li>1. <math>5 + 3 = 8</math> parts</li> <li>2. <math>32 \div 8 = 4</math> (so each part is worth €4)</li> <li>3. <math>5</math> parts = <math>5 \times 4 = \text{€}20</math><br/> <math>3</math> parts = <math>3 \times 4 = \text{€}12</math></li> </ol> | <p>(ii) Multiply each number by 2 to remove the fraction</p> <p style="padding-left: 20px;"><math>1 : 2 : 4</math></p> <ol style="list-style-type: none"> <li>1. <math>1 + 2 + 4 = 7</math> parts</li> <li>2. <math>63 \div 7 = 9</math></li> <li>3. <math>1</math> parts = <math>1 \times 9 = 9</math> cm<br/> <math>2</math> parts = <math>2 \times 9 = 18</math> cm<br/> <math>6</math> parts = <math>4 \times 9 = 36</math> cm</li> </ol> |
|---|---|

#### Example 2:

A and B share a sum of money in the ratio 5 : 3. If A's share is €45, Calculate (i) B's share, (ii) total sum of money

**Answer:**

- (i) A received €45 and had 5 parts of the money
- SO  $5$  parts = 45
- Divide both sides by 5 to get the value of 1 part
- $1$  part =  $45 \div 5$
- $1$  part = 9
- B received 3 parts of the money
- SO.....  $3$  parts =  $3 \times 9 = \underline{27}$
- B got €27.
- (ii) total money = A's share + B's share
- =  $45 + 27$
- = €72

**Percentages:**

Example:

VAT of 21% is added to a bill of €180. Calculate the total bill including the VAT.

Answer:

1. Find 21% of €180

$$180 \times 21 \div 100 = 37.80 \text{ (this is our VAT)}$$

2. Bill exclusive of VAT + VAT added = Bill inclusive of VAT.

$$\begin{array}{rccccccc} \text{€180} & & + & \text{€37.80} & = & & \text{€217.80} \end{array}$$

Example:

12% of a number is 66. What is the number?

Answer:

Write the equation in disguise

$$\rightarrow 12\% = 66$$

Find 1% so divide each side by the number in front of the %

$$\rightarrow 1\% = 66 \div 12$$

$$\rightarrow 1\% = 5.5$$

Now find 100%

$$\rightarrow 100\% = 550$$

So the number is 550.

Example:

A salespersons commission for selling cars is 3½%. If the commission for selling a car was €700, how much did the car cost?

Answer:

Write the equation in disguise

$$\rightarrow 3\frac{1}{2}\% = 700$$

Find 1% so divide each side by the number in front of the %

$$\rightarrow 1\% = 700 \div 3\frac{1}{2}$$

$$\rightarrow 1\% = 200$$

Now find 100%

$$\rightarrow 100\% = 200 \times 100$$

$$100\% = 20,000$$

So the car costs €20,000

**Relative Error:**

Error = true value - estimated value

$$\text{Relative error} = \frac{\text{Error}}{\text{TrueValue}}$$

$$\text{Percentage Error} = \frac{\text{Error}}{\text{TrueValue}} \times 100$$

Example:

A person's weight was estimated to be 90kg. He actually weighed 100kg. Calculate (i) Error (ii) Relative Error (iii) Percentage Error.

Example:

The answer to  $3.4 + 5.3$  was given as 8.5. Calculate the percentage error.

Chapter 11: Arithmetic

**Foreign Exchange:**

Example:

On a certain day,  $\text{€}1 = \text{£}0.70\text{stg}$

Find the value of:

- (i)  $\text{€}300$  in pounds
- (ii)  $\text{£}140$  in euro

Example:

A person bought \$6200 when the exchange rate was  $\text{€}1 = \$1.25$ . A bank charges for this transaction. How much was this charge if the person pays  $\text{€}5000$  for the dollars?

**Compound Interest.**

Compound interest involves money being invested or borrowed over a period of time.

**Some terms.**

**P** = Principal – the money borrowed or invested.

**A** = Amount – the money at the end of the investment.

**R** = Rate – the percentage for the interest

**T** = Time – length of the investment or loan.

$$\text{Principal} \times \text{Rate} = \text{Amount}$$

**NOTE:** The **Amount** at the end of year 1 becomes the **Principal** for year 2 and so on.....

**Example.**

Calculate the compound interest on €3000 for 3 years at 5% per annum.

**Answer:**

From the question: P = 3000      R = 5      T = 3

**Rate:** Firstly use  $\left(1 + \frac{R}{100}\right)$  to help calculate the principal and interest for each year.

$$\left(1 + \frac{R}{100}\right) = \left(1 + \frac{5}{100}\right) = 1.05$$

Year 1:-       $3000 \times 1.05 = 3150$       (3150 = A at end of year 1. This becomes the P for year 2)

Year 2:-       $3150 \times 1.05 = 3307.50$       (3307.50 becomes P for year 3)

Year 3 :-       $3307.50 \times 1.05 = 3472.88$

So in words: €3000 was invested and amounted to €3472.88 at the end of the 3 years.  
So it earned €472.88 interest during the 3 years. ( $3472.88 - 3000 = 472.88$ )  
**Interest = €472.88**

**Questions: Page 227 Q1, 3, 7, 9**

**NOTE:** For depreciation questions use  $\left(1 - \frac{R}{100}\right)$

**Sometimes the interest rate changes every year.****Example:**

€2000 was invested for 3 years at compound interest. The rate for the first year was 10%, the rate for the second year was 8% and for the third year was 9.5%. Calculate the amount and interest earned after the third year.

Rates:- Use  $\left(1 + \frac{R}{100}\right)$        $R_1 = 10\% = 1.1$        $R_2 = 8\% = 1.08$        $R_3 = 9.5\% = 1.095$

Year 1:-       $2000 \times 1.1 = 2200$

Year 2:-       $2200 \times 1.08 = 2376$

Year 3:-       $2376 \times 1.095 = 2601.72$

**Amount = €2601.72**

**Interest earned = 2601.72 – 2000 = 601.72**

**Questions: Page 228 Q11, 13, 14**

**More than 3 years.**

$$A = P \left(1 + \frac{R}{100}\right)^T$$

**Example**

Calculate the compound interest on €23000 for 6 years at 3% per annum.

$A = ?$

$P = 23000$

$R = 3$

$T = 6$

$$A = P \left(1 + \frac{R}{100}\right)^T$$

$$A = 23000 \left(1 + \frac{3}{100}\right)^6$$

$A = 27463.20$

**Amount = €27463.20**

**Interest = 27463.20 – 23000 = €4463.20**

**Questions.**  
**Page 228**  
**Q18, 19, 20, 21**

**Repayments.**

In some questions money is repaid or further money is invested at the end of the first, second or third year. The questions are answered like in compound interest. The investment or repayment is added to or taken of at the end of each year.

**Example:**

A man borrowed €4000 at 8% per annum compound interest. He agreed to repay €1500 at the end of year 1 and €2000 at the end of year 2, and clear the debt at the end of the third year. How much did he pay to clear the debt.

**Answer:**

$$P = 4000 \text{ and } R = 8\% = \left(1 + \frac{R}{100}\right) = 1.08$$

Year 1:  $4000 \times 1.08 = 4320$   
At the end of year 1, €1500 is taken off.

Repayment 1:  $4320 - 1500 = 2820$ .

Year 2:  $2820 \times 1.08 = 3045.6$   
At the end of year 2, €2000 is taken off.

Repayment 2:  $3045.6 - 2000 = 1045.60$

Year 3:  $1045.60 \times 1.08 = 1129.29$

This debt is cleared at the end of year 3, so **€1129.29** has to be paid to clear the debt.

**Questions.**  
**Page 230**  
**Q1, 2, 3**

**Calculating the Rate.**

**Example:**

€8500 was invested for 2 years at compound interest

- (i) The rate of interest for the first year was 4%.  
Find the amount of the investment at the end of the first year.
- (ii) The amount of the investment at the end of the second year was €9237.80.  
Find the rate of interest for the second year.

**Answer:**

$$P = 8500 \quad R = 4\% \text{ or } 1.04$$

(i) Year 1:  $8500 \times 1.04 = 8840$  (at the end of yr 1, the investment is now worth €8840)

(ii) Start of Yr 2 = 8840  
End of Yr 2 = 9237.80

SO; the interest earned in year 2 is €397.80 (923780 – 8840)

$$\text{Rate} = \frac{\text{Interest earned}}{\text{Principal}} \times 100 = \frac{397.80}{8840} \times 100 = 4.5\%$$

**Questions.**  
**Page 231**  
**Q4, 6, 9, 11**

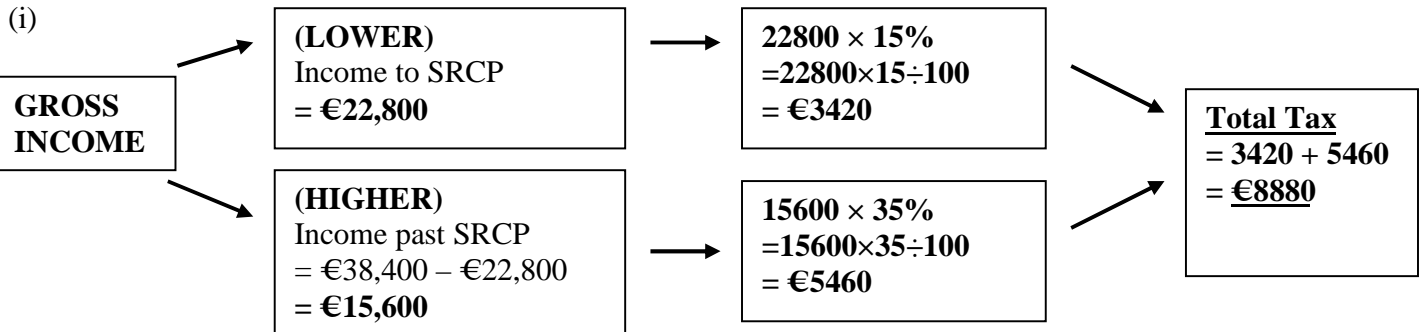
**Income tax**

**Example.**

A person has a gross yearly income of €38,400. They have a standard rate cut-off point of €22,800 and a tax credit of €1,332. The standard rate of tax is 15% of income up to the standard rate cut-off point and 35% on all income above the standard rate cut-off point.

- Calculate (i) gross tax for the year  
 (ii) tax paid for the year  
 (iii) net income for the year

**Answer:**



**Gross Tax = €8,880**

- (ii) Tax Payable / Paid = Gross tax - Tax credits  
 = 8880 - 1,332  
 = 7,548

**Tax Payable / Paid = €7,548**

- (iii) Net income = Gross income - Tax Paid  
 = 38400 - 7548  
 = 30,852

**Net income = €30,852**

**Questions.**  
**Page 234**  
**Q1, 2, 4**

**Calculating the Rate.**

**Example.**

A man has a gross yearly income of €25000. He has a standard rate cut-off point of €29000 and tax credits of €2450. If he pays tax of €2300, calculate his rate of tax.

**Answer:**

Firstly calculate 1) his gross tax and 2) his rate.

**Gross tax.** → Gross tax = Tax paid + Tax credits.  
 = 2300 + 2450 = **4750**

**Rate.** → Rate =  $\frac{\text{Gross Tax}}{\text{Gross Income}} \times 100 = \frac{4750}{25000} \times 100 = \underline{\underline{19\%}}$

**Questions.**  
**Page 234**  
**Q5, 6**

**Index Notation.**

Index notation is used when dealing with very big numbers.

A calculator is required to add, subtract, multiply or divide these numbers.

When dealing with these, we use either **Exp** or  $\times 10^x$  in the calculator.

**Example.**

Express each of the following in the form  $a \times 10^n$  where  $1 \leq a \leq 10$

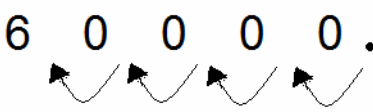
- (i) 60000
- (ii) 7230000
- (iii) 0.00342

**Answer.**

Firstly, move the decimal point until we get a number between 1 and 10.

Count the number of places, that becomes the power.

If it moves to the **left**, it is **positive**, - if it moves to the **right**, it is **negative**.

(i)   $= 6 \times 10^4$  (Decimal point moves 4 places to the left so the power is +4)

(ii)  $7230000 = 7.23 \times 10^6$  (Decimal point moves 6 places to the left to get a number between 1 and 10)

(iii)  $0.00342 = 3.42 \times 10^{-3}$  (Decimal point moves 4 places to the right so power is -3)

**Questions.**  
Page 236  
Q2, 4, 6, 9,  
10, 12

**Example.**

Express the following in the form  $a \times 10^n$  where  $1 \leq a \leq 10$

$$2.52 \times 10^6 + 2.8 \times 10^5$$

**Answer.**

In the calculator, type:

$$2.52 \times 10^6 + 2.8 \times 10^5 \quad \text{or} \quad 2.52 \text{ Exp } 6 + 2.8 \text{ Exp } 5$$

$$= 2800000$$

$$= 2.8 \times 10^6$$

**Questions.**  
Page 238  
Q1, 3, 5, 7, 9,  
11, 13, 15, 17,  
19, 21