Chapter 12

Operations

When we take one operation and follow it by another operation, we must multiply the outcomes.

When we take one operation or a second operation we add the outcomes.

AND = × and OR = +

Permutations

A permutation is an arrangement of a number of objects in a definite order.

Consider 3 people in a race, Tom, Ann and Ben. There are six possible arrangements for these 3 people to finish the race in 1st, 2nd or 3rd. You could have

Tom, Ann, Ben or Tom, Ben, Ann or Ben, Ann, Tom
Ben, Tom, Ann or Ann, Ben, Tom or Ann, Tom, Ben

Another way to do this is to use the box method.
Any of the **three** people could have finished first, however once the first place is taken, there are only **two** people that could finish second. After that there is only **one** person who could have finished last.
So we get : $\boxed{3} \times \boxed{2} \times \boxed{1} = 6$

Example:

(i) How many different arrangements can be made using the letters of the word DUBLIN?
(ii) How many arrangements begin with the letter D?
(iii) How many arrangements begin with B and end with L?
(iv) How many arrangements end with LIN?

Answer:

(i) $\square \times \square \times \square \times \square \times \square \times \square =$

(ii) $\square \times \square \times \square \times \square \times \square \times \square =$

(iii) $\square \times \square \times \square \times \square \times \square \times \square =$

(iv) $\square \times \square \times \square \times \square \times \square \times \square =$

Factorials:

Definition:
The product of all the positive whole numbers from n down to 1 is called "factorial n" and is denoted by $n!$

So, $n! = n(n-1)(n-2) \ldots\ldots \times 3 \times 2 \times 1$.

In other words:
$1! = 1$
$2! = 2 \times 1 = 2$
$3! = 3 \times 2 \times 1 = 6$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

Example:

Evaluate: $\frac{8!}{3!}$

Answer:

$\frac{8!}{3!} = \frac{8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1} = \frac{40230}{6} = 6720$ (in a calculator: $8! \div 3! = 6720$)

Combinations:

A combination is a selection of a number of objects in any order.

$\binom{n}{r}$ notation or ^nC_r

$\binom{n}{r}$ gives the number of ways of choosing r objects from n different objects.

Example:

Evaluate (i) $\binom{9}{2}$ and (ii) $8\binom{5}{2}+5\binom{7}{3}$

Answer:

These questions can be done on a calculator.

(i) $\binom{9}{2}$ = 9 ^nC_r 2 = **36**

(ii) $8\binom{5}{2}+5\binom{7}{3}$ = 8(5 ^nC_r 2) + 5(7 ^nC_r 3)

= 8(10) + 5(35)
= 80 + 175
= **255**

Chapter 12 Probability

Practical Applications of Combinations:

$^{n}C_{r}$ gives the number of ways of choosing *r* objects from *n* different objects.

n = number of different objects we have to choose from.
r = number of objects we choose at one time.

Example 1:
20 people are in a football squad. How many ways can a panel of 11 be chosen.

Answer:
$n = 20$ and $r = 11$
So in the calculator, $^{20}C_{11} = \underline{167,960}$
So there are 167,960 different combinations of picking 11 players from a panel of 20.

Example 2:
In how many ways can a party of 6 children be chosen from a group of 10 if:

(i) any child may be selected
(ii) the oldest child must not be selected
(iii) the youngest child must be selected
(iv) the youngest and oldest must be selected

Answer:
(i) $n = 10$ and $r = 6$
$^{10}C_6 = \underline{210}$

(ii) The oldest child must not be selected so instead of having 10 to choose from, we now have 9.
$n = 9$ and $r = 6$
$^{9}C_6 = \underline{84}$

(iii) The youngest child must be selected so we have 9 children left to choose from and only another 5 children to choose.
$^{9}C_5 = \underline{126}$

(iv) The youngest and oldest must be selected so we have 8 children to choose from and only 4 more to choose.
$^{8}C_4 = \underline{70}$

Practical Applications from Two Different Groups of Objects:

Sometimes we have to deal with problems choosing two objects from two different groups. There are two key words here. **AND means ×** and **OR means +**

Example:
In how many different ways is it possible to choose a group of 4 men and 2 women from 6 men and 5 women.

Answer:
MEN: There are 6 men altogether and we must choose 4.

WOMEN: There are 5 women altogether and we must choose 2.

The question says 4 men **and** 2 women so we must **multiply**.

So……

$${}^{6}C_{4} \times {}^{5}C_{2}$$
$$15 \times 10 = \underline{\mathbf{150}}$$

Example:
A committee of 3 people is selected from a group of 15 doctors and 12 dentists.
In how many different ways can the 3 people be selected
(i) if there are no restrictions
(ii) if the selection must contain exactly 2 doctors
(iii) if the selection must contain at least 1 doctor and at least 1 dentist
(iv) if the selection must contain one specific doctor and one specific dentist?

Answer:
Now remember that n = number of different objects we have to choose from and r = number of objects we choose at one time.

(i) No restrictions so $n = 15 + 12 = 27$, $r = 3$ so…. ${}^{27}C_{3} = \underline{\mathbf{2925}}$

(ii)This has to contain 2 doctors so it must also have 1 dentist.
2 doctors **and** 1 dentist.
$${}^{15}C_{2} \times {}^{12}C_{1} = \underline{\mathbf{1260}}$$

(iii) At least 1 doctor and at least 1 dentist means
1 doctor and 2 dentists or 2 doctors and 1 dentist
$${}^{15}C_{1} \times {}^{12}C_{2} + {}^{15}C_{2} \times {}^{12}C_{1} = \underline{\mathbf{2250}}$$

(iv) 1 specific doctor and 1 specific dentist and 1 other person.
$$1 \times 1 \times {}^{25}C_{1} = \underline{\mathbf{25}}$$

Chapter 12 Probability

Probability

Definition:

The measure of the probability of an event, E, is given by

$$P(E) = \frac{number.of.successful.outcomes}{number.of.possible.outcomes}$$

The probability of an event is a number between 0 and 1.

Example:

In a class, there are 15 boys ad 13 girls. Four boys wear glasses and three girls wear glasses.
A pupil is picked at random from the class.

(i) What is the probability the pupil is a boy?

(ii) What is the probability the pupil wears glasses?

(iii) What is the probability the pupil is a boy who wears glasses?

(iv) A girl is picked at random. What is the probability that she wears glasses?