

Chapter 2 - Complex Numbers

Complex numbers was created to deal with finding the square root of a minus number e.g. $\sqrt{-4}$
This is an **imaginary number** as it does not exist!

$$i = \sqrt{-1} \quad \text{AND} \quad i^2 = -1$$

ALL IMAGINARY NUMBERS ARE EXPRESSED USING THE LETTER i .

Example:

Express the following in terms of i , where $\sqrt{-1} = i$

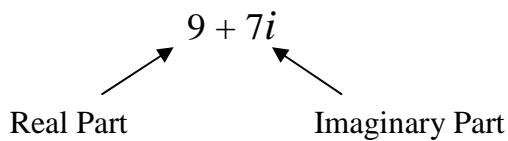
(i) $\sqrt{-16}$

(ii) $\sqrt{-81}$

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Question 1 – 9

Identifying complex numbers.



Example:

Write down the real and imaginary parts of the following.

(i) $4 + 3i$

(ii) $5 - 4i$

(iii) 8

(iv) $2i$

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Question 1 – 12

Adding and Subtracting complex numbers.

Add/Subtract real parts to real parts and imaginary parts to imaginary parts ONLY

Example: Simply the following to the form $a + bi$

(i) $(2 + 3i) + (7 + 6i)$

(ii) $(6 - 2i) - (9 + 5i)$

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Question 1 – 7

Multiplication by a real number.

This is the same as algebra.

Example: Simply the following to the form $a + bi$

(i) $4(1 + 3i) + 2(4 + 6i)$

(ii) $5(2 - 5i) - 3(7 + 4i)$

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Questions 8 – 10

Questions 11 – 19

Multiplication of a Complex Number.

i^2 is replaced with a -1

Example:

Express (i) $(2 + 3i)(4 - 2i)$

(ii) $(2 - i)(-5 + 2i)$

Answer:

$$(2 + 3i)(4 - 2i)$$

Multiply out the brackets like in ALGEBRA

$$8 - 4i + 12i - 6i^2$$

Replace the i^2 with a -1

$$8 - 4i + 12i - 6(-1)$$

$$8 - 4i + 12i + 6$$

Add like terms together!!

$$8 + 6 - 4i + 12i$$

$$14 + 8i$$

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Example 2:

If $z_1 = 4 - 2i$ and $z_2 = -1 + 5i$, find the value of iz_1z_2

Page 33 Question 13 - 20

Complex Conjugate.

The complex conjugate **only differs** from the original by the **sign** of the imaginary part.

If $z = a + bi$ then the complex conjugate $\bar{z} = a - bi$

Note the **+** changed to a **-**

Example:

Find the complex conjugate of the following.

(i) $z = 2 + 9i$

(ii) $z = -5 + 4i$

(iii) $z = 8i$

$\bar{z} =$

$\bar{z} =$

$\bar{z} =$

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Question 1 - 8

Example 2.

Find $z + \bar{z}$ if $z = 3 + 4i$

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Question 9 - 16

Find $z - \bar{z}$ if $z = -2 - 3i$

Find $z \cdot \bar{z}$ if

(i) $z = 2 - 7i$

(ii) $z = -6i$

Division of a Complex Number.

Multiply the top and bottom by the conjugate of the bottom

Example 1.

Express $\frac{2+10i}{3+2i}$ in the form $a + bi$

Example 2.

If $w = 3 - 2i$, express $\frac{1}{w}$ in the form $a + bi$

Equality of Complex Numbers.

To solve these questions we let the **real parts = real parts** and the **imaginary part = imaginary parts**

Example

$$(x + yi) + (3 - i) = 2(1 - 3i) - (y - xi)$$