Chapter 2 - Complex Numbers

Complex numbers was created to deal with finding the square root of a minus number e.g. $\sqrt{-4}$
This is an **imaginary number** as it does not exist!

$\boldsymbol{i = \sqrt{-1}}$ **AND** $\boldsymbol{i^2 = -1}$

ALL IMAGINARY NUMBERS ARE EXPRESSED USING THE LETTER ***i.***

Example:
Express the following in terms of ***i***, where $\sqrt{-1} = i$

(i) $\sqrt{-16}$

(ii) $\sqrt{-81}$

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Question 1 – 9

Identifying complex numbers.

$9 + 7i$

Real Part — Imaginary Part

Example:
Write down the real and imaginary parts of the following.

(i) $4 + 3i$

(ii) $5 - 4i$

(iii) 8

(iv) $2i$

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Question 1 – 12

Adding and Subtracting complex numbers.

Add/Subtract real parts to real parts and imaginary parts to imaginary parts ONLY

Example: Simply the following to the form a + bi

(i) $(2 + 3i) + (7 + 6i)$

(ii) $(6 - 2i) - (9 + 5i)$

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Question 1 – 7

Multiplication by a real number.

This is the same as algebra.

Example: Simply the following to the form a + bi

(i) $4(1 + 3i) + 2(4 + 6i)$

(ii) $5(2 - 5i) - 3(7 + 4i)$

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Questions 8 – 10

Questions 11 – 19

Multiplication of a Complex Number.

i^2 is replaced with a – 1

Example:

Express (i) $(2+3i)(4-2i)$ (ii) $(2-i)(-5+2i)$

Answer:

$(2+3i)(4-2i)$

Multiply out the brackets like in ALGEBRA

$8-4i+12i-6i^2$

Replace the i^2 with a -1

$8-4i+12i-6(-1)$

$8-4i+12i+6$

Add like terms together!!

$8+6-4i+12i$

$14+8i$

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Example 2:

If $z_1 = 4-2i$ and $z_2 = -1+5i$, find the value of iz_1z_2

Page 33 Question 13 - 20

Complex Conjugate.

The complex conjugate **only differs** from the original by the **sign** of the imaginary part.

If $z = a + bi$ then the complex conjugate $\bar{z} = a - bi$

Note the $+$ changed to a $-$

Example:

Find the complex conjugate of the following.

$(i)\ z = 2 + 9i$ $(ii)\ z = -5 + 4i$ $(iii)\ z = 8i$

$\bar{z} =$ $\bar{z} =$ $\bar{z} =$

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Question 1 - 8

Example 2.

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Question 9 - 16

Find $z + \bar{z}$ if $z = 3 + 4i$

Find $z - \bar{z}$ if $z = -2 - 3i$

Find $z.\bar{z}$ if

(i) $z = 2 - 7i$

(ii) $z = -6i$

Division of a Complex Number.

Multiply the top and bottom by the conjugate of the bottom

Example 1.

Express $\frac{2+10i}{3+2i}$ in the form a + bi

Example 2.

If w = 3 – 2i, express $\frac{1}{w}$ in the form a + bi

Equality of Complex Numbers.

To solve these questions we let the real parts = real parts and the imaginary part = imaginary parts

Example

$(x + yi) + (3 – i) = 2(1 – 3i) – (y – xi)$