Chapter 6: Coordinate Geometry of the Circle.

Equation of a circle with centre (0, 0) and radius *r* is $x^2 + y^2 = r^2$

Example 1:
Find the equation of a circle with centre (0, 0) and radius of 7.

Answer.
Centre = (0, 0)
r = 7
So we use the formula $x^2 + y^2 = r^2$

$x^2 + y^2 = r^2$

$x^2 + y^2 = 7^2$

$x^2 + y^2 = 49$

Replace r with a 7

Example 2:
Find the equation of a circle with centre (0, 0) and radius of $\sqrt{3}$.

Answer.
Centre = (0, 0)
r = $\sqrt{3}$
So we use the formula $x^2 + y^2 = r^2$

$x^2 + y^2 = r^2$

$x^2 + y^2 = \sqrt{3}^2$

$x^2 + y^2 = 3$

Example 3:
Find the equation of a circle with centre (0, 0) and contains the point (3, -1)

Answer.
We know the centre but must find the radius. The radius is the distance from the centre (0, 0) to the point on the circle (3, -1)

1) *So use the distance formula.*

$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

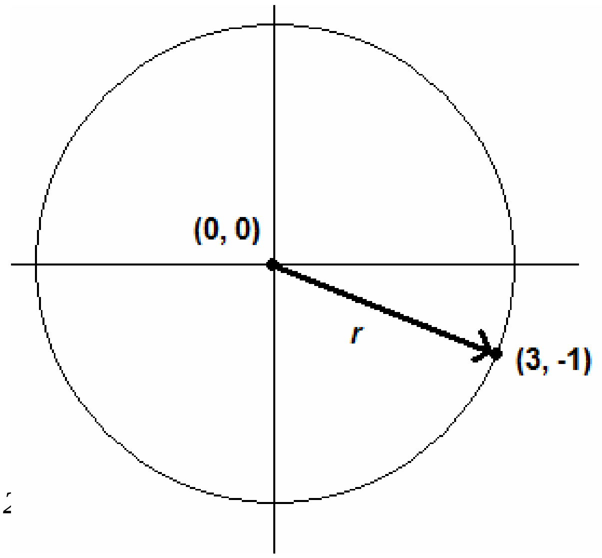
$x_1 = 0 \quad y_1 = 0 \quad x_2 = 3 \quad y_2 = -1$

$\text{Distance} = \sqrt{(3-0)^2 + (-1-0)^2}$

$\text{Distance} = \sqrt{(3)^2 + (-1)^2}$

$\text{Distance} = \sqrt{9+1} = \sqrt{10}$

Radius, r = $\sqrt{10}$



2) *Now find the equation like in example 1 and 2*

$x^2 + y^2 = r^2$

$x^2 + y^2 = \sqrt{10}^2$

$x^2 + y^2 = 10$

Example 4:
Write down the centre and radius of the circle with an equation $x^2 + y^2 = 25$

Answer:
Compare to the equation

$x^2 + y^2 = 25$

$x^2 + y^2 = r^2$ (as you can see, r^2 is equal to 25 so solve the equation)

$r^2 = 25$

$r = \sqrt{25} = 5$

Points Inside, On or Outside the Circle.

To find out if a point is on, inside or outside a circle, we simply substitute the **x** coordinate and **y** coordinate into the general equation.

1. If $x^2 + y^2 = r^2$, then the point is **on** the circle.
2. If $x^2 + y^2 < r^2$, then the point is **inside** the circle.
3. If $x^2 + y^2 > r^2$, then the point is **outside** the circle.

Example 1:

Investigate whether the points (5, 2), (-1, 3) and (2, 0) are on, inside or outside the circle $x^2 + y^2 = 10$

Answer:

(5, 2)
Substitute x = 5 and y = 2 into the equation.

$x^2 + y^2 = 10$

$(5)^2 + (2)^2 = 10$

$25 + 4 = 10$

$29 = 10$

WRONG......

$29 > 10$ so (5, 2) is outside

(-1, 3)
Substitute x = -1 and y = 3 into the equation.

$x^2 + y^2 = 10$

$(-1)^2 + (3)^2 = 10$

$1 + 9 = 10$

$10 = 10$

CORRECT....

So (-1, 3) is on the circle

(2, 0)
Substitute x = 2 and y = 0 into the equation.

$x^2 + y^2 = 10$

$(2)^2 + (0)^2 = 10$

$4 + 0 = 10$

$4 = 10$

WRONG......

$4 < 10$ so (2, 0) is inside

Example 2:

The point (3, k) lies on the circle $x^2 + y^2 = 25$. Find the 2 real values of k.

Answer.

Sub x = 3 and y = k into the equation.

$(3)^2 + (k)^2 = 25$

$9 + k^2 = 25$

$k^2 = 25 - 9$

$k^2 = 16$

$k = \sqrt{16} = 4 \ or \ -4$

Other questions similar to example 2.

1. The point (1, p) lies on the circle $x^2 + y^2 = 10$. Find the 2 real values of p.
2. The point (k, 6) lies on the circle $x^2 + y^2 = 40$. Find the 2 real values of k.
3. The point (-3, s) lies on the circle $x^2 + y^2 = 73$. Find the 2 real values of s.
4. The point (a, -4) lies on the circle $x^2 + y^2 = 41$. Find the 2 real values of a.
5. The point (7, b) lies on the circle $x^2 + y^2 = 170$. Find the 2 real values of b.

General Equation of a Circle.

General Equation of a circle with centre (h, k) and radius *r* is

$$(x-h)^2+(y-k)^2=r^2$$

This equation is used when the centre of the circle is not a (0, 0).

Example 1:
Find the equation of a circle with centre (2, -3) and a radius of 6.

Answer.
The centre is (2, -3) so h = 2 and k = -3
The radius is 6 so $r = 6$

So fill those values into the general equation and simplify

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-2)^2+(y--3)^2=6^2$$

$$(x-2)^2+(y+3)^2=36$$

Example 2: (Exam type question)
Find the equation of a circle that has the line segment [ab] as the diameter, where a=(2, -6) and b=(-4, 4).

Answer.
To find the equation, we need the **centre** and the **radius**.

Finding the centre.
If (2, -6) is one end of the diameter, and (-4, 4) is the other end of the diameter then the mid-point between the two would give the centre of the circle.

Use the mid-point formula.

$$\text{Midpoint}=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$x_1 = 2$, $y_1 = -6$, $x_2 = -4$, $y_2 = 4$.

$$\text{Midpoint}=\left(\frac{2-4}{2},\frac{-6+4}{2}\right)$$

$$\text{Midpoint}=\left(\frac{-2}{2},\frac{-2}{2}\right)=(-1,-1)$$

So the centre = (-1, -1)
Or h = -1 and k = -1.

Finding the radius.
The radius is the distance from the centre to a point on the circle. So we need to find the distance from (-1, -1) to either (2, -6) or (-4, 4)

Using the distance formula.

$$\text{Distance}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

[Using the points (-1, -1) and (2, -6)]
$x_1 = -1$, $y_1 = -1$, $x_2 = 2$, $y_2 = -6$,

$$\text{Distance}=\sqrt{(2--1)^2+(-6--1)^2}$$

$$\text{Distance}=\sqrt{(2+1)^2+(-6+1)^2}$$

$$\text{Distance}=\sqrt{(3)^2+(-5)^2}$$

$$\text{Distance}=\sqrt{34}$$

So radius, $r = \sqrt{34}$

So using these results and the general equation,

$$(x-h)^2+(y-k)^2=r^2$$

$$(x--1)^2+(y--1)^2=\sqrt{34}^2$$

$$(x+1)^2+(y+1)^2=34$$

Finding the Centre and Radius of a Circle.

Example 1:
Find the centre and radius of the circle with an equation of $(x+4)^2 + (y-3)^2 = 49$

Answer.
Compare the given equation to the general equation.

$$(x+4)^2 + (y-3)^2 = 49$$

$$(x-h)^2 + (y-k)^2 = r^2$$

Look at the arrows and write down the equations.

-h = 4   -k = -3 $r^2 = 49$

h = –4   k = 3 $r = \sqrt{49} = 7$

Centre = (h, k) = (–4, 3)
Radius = 7

Points Inside, On or Outside the Circle. (Part 2)

This is done the same as in the normal equation and it follows the same rules!!

Example:
Investigate whether the points (3, 1) and (7, -1) are on, inside or outside the circle $(x-2)^2 + (y+3)^2 = 17$

Answer.
(3, 1):

Sub x = 3 and y = 1.

$(3-2)^2 + (1+3)^2 = 17$

$(1)^2 + (4)^2 = 17$

$1 + 16 = 17$

$17 = 17$

Correct so it is on the circle.

(7, -1)

Sub x = 7 and y = -1

$(7-2)^2 + (-1+3)^2 = 17$

$(5)^2 + (2)^2 = 17$

$25 + 4 = 17$

$29 = 17$

Wrong!!
$29 > 17$
So it is outside the circle!!

Equation of a Tangent to a circle at a Given Point.

Method:

1. Find the slope
2. Turn the slope upside-down and change the sign. (This gives the slope of the tangent)
3. Use the slope and point of contact in the Equation formula.

$$(y - y_1) = m(x - x_1)$$

Example:

Find the equation of the tangent to the circle $x^2 + y^2 = 8$ at the point (2, 2) on the circle.

Answer:

Step 1: Slope of the radius, using the points (0, 0) and (2, 2)

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2-0}{2-0} = \frac{2}{2} = 1$$

Step 2: Slope = 1, so turn upside down and change the sign = -1

Step 3:

$x_1 = 2 \quad y_1 = 2 \quad m = -1$

$(y - y_1) = m(x - x_1)$

$(y - 2) = -1(x - 2)$

$y - 2 = -x + 2$

Bring everything to the left hand side.(remember to change signs!!)

$x + y - 2 - 2 = 0$

$x + y - 4 = 0$

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Example 2:

Find the equation of the tangent to the circle $(x+2)^2 + (y-3)^2 = 29$ at the point (3, 5) on the circle.

Answer:

Centre =(h, k) = (-2, 3)

So find the slope between the points (-2, 3) and (3, 5)

Step 1:

$x_1 = -2 \quad y_1 = 3 \quad x_2 = 3 \quad y_2 = 5$

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5-3}{3--2} = \frac{5-3}{3+2} = \frac{2}{5}$$

Step 2

Now, turn the slope upside down and change the sign

$$\frac{2}{5} = -\frac{5}{2}$$

Step 3:

$x_1 = 3 \quad y_1 = 5 \quad m = -\frac{5}{2}$

$(y - y_1) = m(x - x_1)$

$(y - 5) = -\frac{5}{2}(x - 3)$

$2(y - 5) = -5(x - 3)$

$2y - 10 = -5x + 15$

Bring everything to the left hand side.

(remember to change signs!!)

$5x + 2y - 10 - 15 = 0$

$5x + 2y - 25 = 0$

Points of Intersection.

When you are asked to find the point of intersection between a circle and a line, we use the simultaneous equations method from Algebra.

Example:

Find the point of intersection between the line $2x+y+10=0$ and the circle $x^2+y^2=40$

Answer:

Step 1: Get x or y on its own.

$2x+y+10=0$

$y=-2x-10$

Step 2: Sub $y=-2x-10$ in for y.

$x^2+y^2=40$

$x^2+(-2x-10)^2=40$

$(-2x-10)^2$

$(-2x-10)(-2x-10)$

$4x^2+20x+20x+100$

$4x^2+40x+100$

$x^2+4x^2+40x+100=40$

$x^2+4x^2+40x+100-40=0$

$5x^2+40x+60=0$

Divide by 5 to simplify.

$x^2+8x+12=0$

Step 3: Solve the quadratic.

$x^2+8x+12=0$

$12=12\times1$ or 6×2 or 4×3

$(x+6)(x+2)=0$

$x+6=0 \qquad x+2=0$

$x=-6 \qquad x=-2$

Step 4: Sub the values from step 3 into the answer from step 1.

$y=-2x-10$

When $x=-6$ $\quad y=-(-6)-10$

$y=6-10$

$y=-4$

When $x=-2$ $\quad y=-(-2)-10$

$y=2-10$

$y=-8$

So the points are (-6, -4) and (-2, -8)